DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

23-0009-AQ

TEST BOOKLET
MATHEMATICS
PAPER – I

Time Allowed: 3 hours | **Maximum Marks: 300**

INSTRUCTIONS TO CANDIDATES

Read the instructions carefully before answering the questions: -

1. This Test Booklet consists of **24 (twenty four)** pages and has **75 (seventy-five) items (questions).**
2. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS BOOKLET *DOES NOT* HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
3. **Please note that it is the candidate's responsibility to fill in the Roll Number and other required details carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet and the Separate Answer Booklet. Any omission/discrepancy will render the OMR Answer Sheet and the Separate Answer Booklet liable for rejection.**
4. Do not write anything else on the OMR Answer Sheet except the required information. Before you proceed to mark in the OMR Answer Sheet, please ensure that you have filled in the required particulars as per given instructions.
5. Use **only Black Ball Point Pen** to fill the OMR Answer Sheet.
6. This Test Booklet is divided into 4 (four) parts – **Part – I, Part – II, Part - III** and **Part – IV.**
7. All three parts are **Compulsory.**
8. **Part-I consists of Multiple Choice-based Questions.** The answers to these questions have to be marked in the **OMR Answer Sheet** provided to you.
9. **Part-II, Part-III and Part-IV consist of Conventional Essay-type Questions.** The answers to these questions have to be written in the separate **Answer Booklet** provided to you.
10. In Part-I, each item (question) comprises of 04 (four) responses (answers). You are required to select the response which you want to mark on the OMR Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose *ONLY ONE* response for each item.
11. After you have completed filling in all your responses on the OMR Answer Sheet and the Answer Booklet(s) and the examination has concluded, you should hand over to the Invigilator *only the OMR Answer Sheet and the Answer Booklet(s)*. You are permitted to take the Test Booklet with you.
12. **Penalty for wrong answers in Multiple Choice-based Questions:**

THERE WIL BE **PENALTY** FOR WRONG ANSWERS MARKED BY A CANDIDATE.

(i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to the question will be deducted as penalty.

(ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to the question.

(iii) If a question is left blank. i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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PART-I
(Multiple Choice-based Questions)

Instructions for Questions 1 to 50:

- ***Choose the correct answers for the following questions.***
- ***Each question carries 3 marks.***
- ***No Data Books/Tables are allowed; assume the data if required anywhere.***

[3 x 50 = 150]

1. Every non-zero square matrix $\boldsymbol{A}$ can be expressed as the sum of some symmetric matrix $\boldsymbol{B}$ and some skew-symmetric matrix $\boldsymbol{C}$. If the matrix $\boldsymbol{A}$ is $\begin{bmatrix} 2 & 3 \\ 7 & 8 \end{bmatrix}$, then B and C (respectively) are:

   (a) $\begin{bmatrix} -2 & 5 \\ 5 & 8 \end{bmatrix}, \begin{bmatrix} 4 & -2 \\ 2 & 0 \end{bmatrix}$

   (b) $\begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix}, \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

   (c) $\begin{bmatrix} 2 & 8 \\ 8 & 2 \end{bmatrix}, \begin{bmatrix} 0 & -2 \\ 2 & 6 \end{bmatrix}$

   (d) $\begin{bmatrix} 8 & 2 \\ 2 & 3 \end{bmatrix}, \begin{bmatrix} -6 & 1 \\ -1 & 5 \end{bmatrix}$

2. Given that $B = \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix}$, find a one non-zero vector $U = \begin{bmatrix} x \\ y \end{bmatrix}$ such that BU = 6U.

   (a) $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$

   (b) $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$

   (c) $\begin{bmatrix} -3 \\ 5 \end{bmatrix}$

   (d) $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$

3. Let $\boldsymbol{W}$ be the subspace of all symmetric matrices in $\boldsymbol{M_{2\times2}}$. Then the dimension of $\boldsymbol{W}$ is:

   (a) 1

   (b) 2

   (c) 3

   (d) 4

4. The eigen vector of the matrix $\begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$ associated with the eigen value $\boldsymbol{\lambda} = \mathbf{3}$ is:

(a) $\begin{bmatrix} k \\ 2k \end{bmatrix}$

(b) $\begin{bmatrix} 2k \\ k \end{bmatrix}$

(c) $\begin{bmatrix} -k \\ 2k \end{bmatrix}$

(d) $\begin{bmatrix} 2k \\ -k \end{bmatrix}$

5. Which of the following is a linear transformation?

(a) $T: R \rightarrow R\ s.t.\ T(x) = x^2$

(b) $T: R \rightarrow R\ s.t.\ T(x) = x + 1$

(c) $T: R \rightarrow R\ s.t.\ T(x) = \sin x$

(d) $T: R^2 \rightarrow R^2\ s.t.\ T(x, y) = (x + y, x)$

6. The kernel of the linear transformation $\boldsymbol{T}: \boldsymbol{R}^3 \rightarrow \boldsymbol{R}^2$ defined by $\boldsymbol{T}(\boldsymbol{x}) = \boldsymbol{AX},$ where $A = \begin{bmatrix} 1 & -1 & -2 \\ -1 & 2 & 3 \end{bmatrix}$, is:

(a) (t, t, –t)

(b) (t, –t, t)

(c) (–t, t, t)

(d) (t, t, t)

7. Find the rank and nullity (respectively) of the linear transformation $\boldsymbol{T}: \boldsymbol{R}^3 \rightarrow \boldsymbol{R}^3$ defined by the matrix $A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

(a) 1 and 2

(b) 2 and 1

(c) 3 and 0

(d) 0 and 3

8. The standard matrix for the linear transformation $\boldsymbol{T}: \boldsymbol{R}^3 \to \boldsymbol{R}^2$ defined by $\boldsymbol{T}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = (\boldsymbol{x} - \boldsymbol{2y}, \boldsymbol{2x} + \boldsymbol{y})$ is:

(a) $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 0 \end{bmatrix}$

9. If A = $\begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 5 & 3 & -3 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then which of the following is correct?

(a) $A^3 - 6A^2 + 11A - 6I = 0$
(b) $A^3 - 7A + 6I = 0$
(c) $A^3 - 7A^2 + 6I = 0$
(d) $A^3 + 7A - 6I = 0$

10. Which of the following functions is continuous at $\boldsymbol{x} = \boldsymbol{0}$?

(a) $f(x) = \frac{1}{x}$

(b) $f(x) = \frac{|x|}{x}$

(c) $f(x) = |x|$

(d) $f(x) = \cos\frac{1}{x}$

11. If the function $\boldsymbol{f(x) = (1 + x)^{\cot x}}$ is continuous at $\boldsymbol{x = 0}$, then $\boldsymbol{f(0)}$ will be:

(a) 0

(b) 1

(c) $\frac{1}{e}$

(d) e

12. $\boldsymbol{f(x)}$ is a differentiable function such that $\boldsymbol{f(0) = f(1) = 0}$ and $\boldsymbol{f'(1) = 2}$. If $\boldsymbol{y(x) = f(e^x)e^{f(x)}}$ then $\boldsymbol{y'(0)}$ is:

(a) 0
(b) 1
(c) 2
(d) 2e

13. Let $y = f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x) = \sin x^2$. Then $\frac{dy}{dx}$ at $x = 0$ is equal to:

(a) $\frac{1}{2}\sin 1$

(b) $\sin 1$

(c) $2 \sin 1$

(d) 2

14. If $z = f(y/x^2)$, then $x\frac{\partial z}{\partial x} + 2y\frac{\partial z}{\partial y}$ is:

(a) $\frac{2y}{x^2} f'(y/x^2)$

(b) $\frac{4y}{x^2} f'(y/x^2)$

(c) $\frac{4y}{x^3} f'(y/x^2)$

(d) 0

15. If $x = r\cos\theta$, $y = r\sin\theta$, then $\frac{\partial(r,\theta)}{\partial(x,y)}$ is equal to:

(a) $\frac{1}{r^2}$

(b) 1

(c) r

(d) $\frac{1}{r}$

16. The value of the integral $\int_{x=0}^{1}\int_{y=x}^{1} e^{x/y} dx\, dy$ is:

(a) $\frac{1}{2}(e+1)$

(b) $\frac{1}{2}(e-1)$

(c) $e - 1$

(d) $2(e - 1)$

17. The coordinates of the foot of the perpendicular drawn from $(\mathbf{0}, \mathbf{0}, \mathbf{0})$ to the plane $\mathbf{2x} +$ $\mathbf{3y} - \mathbf{4z} + \mathbf{1} = \mathbf{0}$ are:

(a) $\left(\frac{-2}{29}, \frac{-3}{29}, \frac{4}{29}\right)$

(b) $\left(\frac{2}{29}, \frac{3}{29}, \frac{-4}{29}\right)$

(c) $\left(\frac{-2}{3}, -1, \frac{4}{3}\right)$

(d) $\left(\frac{2}{3}, -1, \frac{-4}{3}\right)$

18. The equation of the straight line passing through the origin and parallel to the planes $\mathbf{2x} +$ $\mathbf{3y} + \mathbf{4z} = \mathbf{5}$ and $\mathbf{3x} + \mathbf{4y} + \mathbf{5z} = \mathbf{6}$, is:

(a) $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$

(b) $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$

(c) $\frac{x}{-1} = \frac{y}{2} = \frac{z}{1}$

(d) $\frac{x}{1} = \frac{y}{-2} = \frac{z}{1}$

19. The straight lines $\boldsymbol{x = ay + b}$, $\boldsymbol{z = cy + d}$ and $\boldsymbol{x = a'y + b'}$, $\boldsymbol{z = c'y + d'}$ will be perpendicular to each other if:

(a) $aa' + cc' = 1$
(b) $aa' + cc' + 1 = 0$
(c) $ac' + a'c = 1$
(d) $ac' + a'c + 1 = 0$

20. The coordinates of the point of intersection of the straight lines $\frac{x-3}{1} = \frac{y+4}{-3} = \frac{z-5}{3}$ and $\frac{x-4}{1} = \frac{y-5}{3} = \frac{z+6}{-4}$ are:

(a) (4, –7, 8)
(b) (– 4, 7, –8)
(c) (2, –1, 2)
(d) (–2, 1, –2)

21. A plane passing through the fixed point $(\mathbf{1},\mathbf{1},\mathbf{1})$ meets the coordinate axes at points $\boldsymbol{A},\boldsymbol{B}$ and $\boldsymbol{C}$ respectively. The locus of the centre of the sphere $\boldsymbol{OABC}$ ($\boldsymbol{O}$ being the origin) will be:

(a) $x^{-1}+y^{-1}+z^{-1}=2$

(b) $x^{-1}+y^{-1}+z^{-1}=1$

(c) $x^{-2}+y^{-2}+z^{-2}=1$

(d) $x^{-2}+y^{-2}+z^{-2}=2$

22. If the tangent plane to the sphere $\boldsymbol{x^2+y^2+z^2=r^2}$ makes intercepts at $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ on the coordinate axes, then:

(a) $a^2+b^2+c^2=r^{-2}$

(b) $a^{-2}+b^{-2}+c^{-2}=r^{-4}$

(c) $a^{-2}+b^{-2}+c^{-2}=r^2$

(d) $a^{-2}+b^{-2}+c^{-2}=r^{-2}$

23. The perpendicular drawn from the origin to the tangent planes to the cone $\boldsymbol{ax^2+by^2+cz^2=0}$ lies on the surface:

(a) $\frac{a}{x^2}+\frac{b}{y^2}+\frac{c}{z^2}=0$

(b) $\frac{a}{x^2}+\frac{b}{y^2}+\frac{c}{z^2}=1$

(c) $\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}=0$

(d) $\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}=1$

24. The cone $\boldsymbol{ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0}$ will have three mutually perpendicular generators if:

(a) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$

(b) $a+b+c=0$

(c) $\frac{1}{f}+\frac{1}{g}+\frac{1}{h}=0$

(d) $af+bg+ch=0$

25. The equation of the normal to the conicoid $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{36}=1$ at the point (2, 3, 6) is:

(a) $\frac{x-2}{3}=\frac{y-3}{2}=\frac{z-6}{1}$

(b) $\frac{x-2}{-3}=\frac{y-3}{2}=\frac{z-6}{1}$

(c) $\frac{x-2}{3}=\frac{y-3}{-2}=\frac{z-6}{1}$

(d) $\frac{x-2}{3}=\frac{y-3}{2}=\frac{z-6}{-1}$

26. The equation of the plane that intersects the surface $2x^2-3y^2+5z^2=1$ in a conic having the centre at the point (2, 1, 3) is:

(a) $4x-3y+15z=50$

(b) $4x+3y-15z=50$

(c) $4x-3y+15z=49$

(d) $4x+3y-15z=49$

27. If $L\left(\frac{1}{t}\sin t\right)=\tan^{-1}\left(\frac{1}{s}\right)$, then $L\left(\frac{1}{t}\sin 5t\right)$ is:

(a) $5\tan^{-1}\left(\frac{5}{s}\right)$

(b) $\tan^{-1}\left(\frac{5}{s}\right)$

(c) $\frac{1}{5}\tan^{-1}\left(\frac{5}{s}\right)$

(d) $\frac{1}{5}\tan^{-1}(5s)$

28. The inverse Laplace transform $L^{-1}\left\{\frac{s}{2s^2-8}\right\}$ is equal to:

(a) $2\cosh 2t$

(b) $\frac{1}{2}\sinh 2t$

(c) $2\cosh 4t$

(d) $\frac{1}{2}\cosh 2t$

29. The solution of the differential equation $2x^2 \frac{dy}{dx} = xy + y^2$ is:

(a) $1 + \frac{y}{x} = c\sqrt{x}$

(b) $1 - \frac{y}{x} = c\sqrt{x}$

(c) $1 + \frac{x}{y} = c\sqrt{x}$

(d) $1 - \frac{x}{y} = c\sqrt{x}$

30. If $\frac{1}{x}(c_1 + c_2 \log x)$ is the general solution of the differential equation $x^2 \frac{d^2y}{dx^2} - kx\frac{dy}{dx} + y = 0;\ x > 0$ then **k** equals to:

(a) -3

(b) -1

(c) 2

(d) 3

31. Solution of the differential equation $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is given by:

(a) $y = sinh^{-1}(C_1 + x) + C_2$

(b) $y = cosh(C_1 + x) + C_2$

(c) $y = sinh(C_1 + x) + C_2$

(d) $y = cosh^{-1}(C_1 + x) + C_2$

32. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solutions of the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-4y=0$. If $W(x)$ is the Wronskian of the solutions of the differential equation, then $W(3)-W(2)$ is equal to:

(a) $\frac{2}{3}$

(b) $\frac{10}{3}$

(c) $\frac{2}{3}$

(d) $\frac{10}{3}$

33. If $D\equiv\frac{d}{dx}$, then the particular solution $\left[\frac{1}{xD+1}\right]x^{-1}$ is:

(a) $\log x$

(b) $\frac{1}{x}\log x$

(c) $\frac{1}{x^2}\log x$

(d) $-x\log x$

34. If e^{-2x} and $\sin 3x$ are known to be two solutions of the homogeneous linear differential equation of order three with constant coefficients, then the differential equation must be:

(a) $\frac{d^3y}{dx^3}+11\frac{d^2y}{dx^2}+18y=0$

(b) $\frac{d^3y}{dx^3}-7\frac{d^2y}{dx^2}-18y=0$

(c) $\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}+9\frac{dy}{dx}+18y=0$

(d) $\frac{d^3y}{dx^3}-2\frac{d^2y}{dx^2}+9\frac{dy}{dx}-18y=0$

35. A horizontal shelf is moving up and down with SHM of period $\frac{1}{2}$ second. What should the amplitude of the motion be such that a book placed on the shelf may not fall off?

(a) $\frac{\pi^2 g}{16}$

(b) $16\pi^2$

(c) $4\pi^2$

(d) $16\pi^2 g$

36. A particle is projected with initial velocity of projection $\boldsymbol{u}$ and angle of projection $\boldsymbol{\alpha}$. If the equation of the projectile is given by $\boldsymbol{y = x - x^2}$, then the velocity of projection is:

(a) $\sqrt{2g}$

(b) $\frac{1}{2}\sqrt{2g}$

(c) $\sqrt{g}$

(d)

37. A heavy particle of weight $\boldsymbol{W}$ attached to a fixed point by a light inextensible chord, describes a full circle in a vertical plane. The tensions in the chord being $\boldsymbol{mW}$ and $\boldsymbol{nW}$ respectively when the particle is at the highest and the lowest points in the path. Then, $\boldsymbol{m}$ and $\boldsymbol{n}$ satisfy the relation:

(a) $m = n + 6$

(b) $n = m + 6$

(c) $m = n + 4$

(d) $n = m + 4$

38. A particle describes an equiangular spiral $\boldsymbol{r = ae^{\theta \cot \alpha}}$ under a central force $\boldsymbol{F}$ towards the pole. The law of force is:

(a) $P \propto kr^2$

(b) $P \propto kr^{-5}$

(c) $P \propto kr^{-3}$

(d) $P \propto kr^{-2}$

39. The periodic time of a planet moving under inverse square law of acceleration is:

(a) $\pi\sqrt{a^3/\mu}$

(b) $\frac{2\pi a}{\sqrt{\mu}}$

(c) $\pi\sqrt{a/\mu}$

(d) $2\pi\sqrt{a^3/\mu}$

40. If the ratio of the major axes of the elliptic orbits of two planets is $\frac{4}{9}$, then the ratio of their periodic times is equal to:

(a) 2 : 3
(b) 4 : 9
(c) 8 : 27
(d) $\sqrt{2} : \sqrt{3}$

41. If the coefficient of friction is $\frac{1}{\sqrt{3}}$, then the height of the particle that can rest inside a rough hollow sphere of radius a is equal to:

(a) $\frac{3a}{2}$

(b) $\frac{a}{2}$

(c) $\left(\frac{2+\sqrt{3}}{2}\right)a$

(d) $\left(\frac{2-\sqrt{3}}{2}\right)a$

42. If $f(r)$ is a function of r, where $r^2 = x^2 + y^2 + z^2$, then $\nabla^2 f(r)$ is equal to:

(a) $\frac{d^2f}{dr^2} - \frac{1}{r}\frac{df}{dr}$

(b) $\frac{d^2f}{dr^2} + \frac{1}{r^2}\frac{df}{dr}$

(c) $\frac{d^2f}{dr^2} + \frac{2}{r}\frac{df}{dr}$

(d) $\frac{d^2f}{dr^2} \quad \frac{2}{r^2}\frac{df}{dr}$

43. If the function $\emptyset(x, y, z)$ is a solution of the Laplace's equation, then the vector function $\nabla\emptyset$ is ________

(a) a non-zero constant vector.
(b) Solenoidal.
(c) irrotational.
(d) both solenoidal and irrotational.

44. The radius of curvature of the curve $x = 3\cos t, y = 3\sin t, z = 4t$ at the point 't' is:

(a) $\frac{3}{25}$
(b) $\frac{25}{3}$
(c) $\frac{4}{25}$
(d) $\frac{25}{4}$

45. If $\hat{r}$ is the unit vector in the direction of the vector $\bar{r}$, then $\hat{r} \times d\hat{r}$ is equal to:

(a) $\frac{\bar{r} \times d\hat{r}}{r^2}$
(b) $\frac{\bar{r} \times d\bar{r}}{r^2}$
(c) $\frac{\hat{r} \times d\bar{r}}{r^2}$
(d) $\frac{\bar{r} \times d\bar{r}}{r^3}$

46. A vector function $\bar{f}$ is the scalar product of a scalar function ϕ and the gradient of another scalar function Ψ. Then $\bar{f}.\text{curl}\,\bar{f}$ is equal to:

(a) $\nabla\phi \cdot \nabla\Psi$
(b) $\text{div}(\nabla\phi \times \text{curl}(\nabla\Psi))$
(c) $\text{div}(\nabla\phi \times \nabla\Psi)$
(d) 0

47. If S is the closed surface of the cube bounded by the planes x = y = z = 0 and x = y = z = a, then the double integral $\int\int_S (x\hat{i} + y\hat{j} + z\hat{k}) \cdot \hat{n}\, ds$ is:

(a) $3a^3$
(b) $2a^3$
(c) a^3
(d) $\frac{1}{3}a^3$

48. If C is the boundary of the rectangle $0 \le x \le \pi,\ 0 \le y \le 1, z = 0$, then the line integral $\int_C (\sin z\ dx - \cos x\ dy + \sin y\ dz)$ is equal to:

(a) -1
(b) 0
(c) 1
(d) 2

49. If C is the circle defined by $x^2 + y^2 = 1$, then the integral

$$\int_C [(\cos x \sin y - xy)dx + \sin x \cos y\, dy]$$

is equal to:

(a) 0

(b) $-\frac{}{3}$

(c) $\frac{1}{3}$

(d) 2

50. The area R in a plane region bounded by a simple closed curve C can be expressed as:

(a) $\frac{1}{2}\int_C (xdy - ydx)$

(b) $2\int_C (xdy - ydx)$

(c) $2\int_C (xdy + ydx)$

(d) $\frac{1}{2}\int_C (xdy + ydx)$

PART-II

Instructions for Questions 51 to 63:

- ***Answer any 10 (TEN) out of the thirteen questions.***
- ***Each question carries 5 marks.***
- ***No Data Books/Tables are allowed; assume the data if required anywhere.***
- ***Unless otherwise mentioned, symbols and notations have their usual meaning.***

[5 x 10 = 50]

51. Determine whether the polynomials $f(x) = 2x^3 + x^2 + x + 1$, $g(x) = x^3 + 3x^2 + x - 2$ and $h(x) = x^3 + 2x^2 - x + 3$ in vector space **R[x]** of all polynomials over the field of real numbers are linearly independent or not.

52. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 1 \end{bmatrix}$, then find two non-singular matrices P and Q such that PAQ = I.

53. Evaluate the following limit:

$$\lim_{x \to -\infty} \frac{x^2(\sin x + \cos^3 x)}{(x^2+2)(x-4)}$$

54. If $u = ze^{ax+by}$, where z is a homogenous function in x and y of degree n, then prove that:

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = (ax + by + n)u.$$

55. A plane passes through a fixed point (p, q, r) and cuts the coordinate axes at points A, B and C. Show that the locus of the centre of the sphere $OABC$ is $\frac{p}{x} + \frac{q}{y} + \frac{r}{z} = 2.$

56. Tangent planes are drawn to the conicoid $ax^2 + by^2 + cz^2 = 1$ through the point $(\alpha,\ \beta,\ \gamma)$. Show that the perpendicular from the centre of the conicoid to these planes generates the cone $(\alpha x + \beta y + \gamma z)^2 = \frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c}$.

57. Solve the differential equation:

$$\left(x\cos\frac{y}{x} + y\sin\frac{y}{x}\right)y - \left(y\sin\frac{y}{x} - x\cos\frac{y}{x}\right)x\frac{dy}{dx} = 0$$

58. Solve the differential equation:

$$p^3 - 4xyp + 8y^2 = 0.$$

59. Find the inverse Laplace transform of the function:

$$\frac{6s-4}{s^2-4s+20}$$

60. At the end of three successive seconds the distances of a particle moving with SHM from its mean position measured in the same direction are **1, 5** and **5** respectively. Prove that the period of complete oscillation is $\boldsymbol{2\pi/\cos^{-1}(3/5)}$.

61. Four light rods are joined together to form a quadrilateral $\boldsymbol{OABC}$. The lengths of the rods are such that $\boldsymbol{OA = OC = a}$ and $\boldsymbol{AB = CB = b}$. The frame-work hangs in a vertical plane with $\boldsymbol{OA}$ and $\boldsymbol{OC}$ resting in contact with two smooth pegs distance $\boldsymbol{l}$ apart and on the same horizontal level. A weight $\boldsymbol{W}$ hangs at $\boldsymbol{B}$. If $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$ are the inclinations of $\boldsymbol{OA}$ and $\boldsymbol{AB}$ to the horizontal, then prove that:

(a) $\boldsymbol{a\cos\theta = b\cos\phi}$ and

(b) $\boldsymbol{\frac{1}{2}l\sec^2\theta\sin\phi = a\sin(\theta+\phi)}$

62. If $\bar{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ is any non-zero vector function, then prove that:

$$\nabla\times(\nabla\times\bar{A}) = \nabla(\nabla\cdot\bar{A}) - \nabla^2\bar{A}$$

63. If $\bar{A} = N\hat{i} - M\hat{j}$, then show that the formula in the Green's theorem in plane may be written as

$$\int\int_R \text{div}\,\bar{A}\,dxdy = \int_C \bar{A}\cdot\hat{n}\,ds,$$

where $\hat{n}$ is the outward drawn unit normal vector to the curve **C** bounding the region **R** and **s** is the arc length of **C**.

PART-III

Instructions for Questions 64 to 71:

- *Answer any 5 (FIVE) out of the eight questions.*
- *Each question carries 10 marks.*
- *No Data Books/Tables are allowed; assume the data if required anywhere.*
- *Unless otherwise mentioned, symbols and notations have their usual meaning.*

[10 x 5 = 50]

64. Let $\boldsymbol{L} : \boldsymbol{R}^3 \rightarrow \boldsymbol{R}^2$ be the linear transformation defined by $\boldsymbol{L}\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y \\ y-z \end{bmatrix}$

Let $\boldsymbol{S} = \{v_1, v_2, v_3\}$ and $\boldsymbol{T}\ \{w_1, w_2\}$ be the bases for $\boldsymbol{R}^3$ and $\boldsymbol{R}^2$ respectively, where –

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, w_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, w_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Find the matrix of $\boldsymbol{L}$ with respect to $\boldsymbol{S}$ and $\boldsymbol{T}$.

65. Show that the function $u = x^3y^2(1-x-y)$ attains its maximum value at the point $\left(\frac{1}{2}, \frac{1}{3}\right)$. Also, find its maximum value.

66. Evaluate the triple integral $\iiint_V f(x,y,z)dV$, where $\boldsymbol{V}$ is the hemisphere with centre at origin and radius $\mathbf{1}$, lying above the plane $z = 0$ and $f(x,y,z) = z^3\sqrt{x^2+y^2+z^2}$.

67. The section of a cone with vertex $\boldsymbol{P}$ and guiding curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$ by the plane $x = 0$ is a rectangular hyperbola. Show that the locus of P is $\frac{x^2}{a^2} + \frac{y^2+z^2}{b^2} = 1$.

68. Solve the following differential equation:

$$x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right).$$

69. Solve the following differential equation:

$$(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x).$$

70. A particle of mass m is placed at the highest point of a smooth vertical circle of radius a and is allowed to slide down starting with a negligible velocity. Prove that it will leave the circle and subsequently describe a parabola of latus rectum $\frac{16}{27}a$

71. Evaluate $\iint_S \bar{F}.\hat{n}\, dS$, where $\bar{F} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and $\boldsymbol{S}$ is the surface of the plane $\mathbf{2x + 3y + 6z = 12}$ in the first octant.

PART-IV

Instructions for Questions 72 to 75:

- *Answer any 2 (TWO) out of the four questions.*
- *Each question carries 25 marks.*
- *No Data Books/Tables are allowed; assume the data if required anywhere.*
- *Unless otherwise mentioned, symbols and notations have their usual meaning.*

[25 x 2 = 50]

72.

(a) Find all the eigenvalues and the associated eigenvectors of the matrix –

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

(b) Show that the set $\boldsymbol{S} = \{\boldsymbol{t^2+1}, \boldsymbol{t-1}, \boldsymbol{2t+2}\}$ forms a basis for the vector space $\boldsymbol{P2}$.

73.

(a) If $u_1 = \frac{x_2x_3}{x_1}$, $u_2 = \frac{x_1x_3}{x_2}$ and $u_3 = \frac{x_1x_2}{x_3}$ then find the Jacobian $\boldsymbol{J(u_1, u_2, u_3)}$.

(b) Find the equation of the right circular cylinder, which passes through the circle

$$\boldsymbol{x^2+y^2+z^2=9,\ x-y+z=3.}$$

74.

(a) Use method of variation of parameters to solve the following differential equation:

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^2e^x.$$

(b) A particle under a central force $\boldsymbol{\mu[2(a^2+b^2)u^5-3a^2b^2u^7]}$ is projected at a distance $\boldsymbol{a}$ with velocity $\boldsymbol{\sqrt{\mu}/a}$ in a direction at right angles to the initial distance. Prove that the path is the curve $\boldsymbol{r^2=a^2\cos^2\theta+b^2\sin^2\theta}$.

75. Verify the Gauss divergence theorem for the function $\overline{F} = y\hat{i}+x\hat{j}+z^2\hat{k}$ over the cylindrical region bounded by the surfaces $\boldsymbol{x^2+y^2=a^2, z=0, z=h}$.

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